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LMI-based robust adaptive synchronization of FitzHugh–Nagumo neurons with unknown parameters under uncertain external electrical stimulation

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ABSTRACT

This Letter addresses a nonlinear robust adaptive control that utilizes linear matrix inequalities for asymptotic synchronization of two coupled chaotic FitzHugh–Nagumo neurons under unknown parameters and uncertain stimulation current amplitudes and phase shifts. Synchronization of chaotic neurons using the proposed control method through numerical simulation is demonstrated.

1. Introduction

Synchronization of chaotic neurons under external electrical stimulation (EES) (for example, deep brain stimulation) has attracted increasing research attention over the past decade in understanding neural system functions and of improving outcomes of external therapies for cognitive disorders [1–3]. Neuronal synchronization, in enabling coordination between different areas of the brain, plays an important role in neural signal transmission. The FitzHugh–Nagumo (FHN) neuron model has been intensively studied and extensively employed as a synchronization-investigation tool owing to its utility in representing neuronal behavior under sinusoidal EES [4].

Various FHN neuron studies concerning chaos and its control, noise effects and filtering as well as tracking and synchronization have been carried out [5–11]. The effects of the frequency of the stimulation current on neural dynamics (for example, the chaotic behavior of FHN neurons under certain frequencies) also have been investigated [4]. Some dynamical studies [12–14] show that the synchronization of identical coupled FHN neurons under EES can be achieved for a sufficiently large gap junction conductance. Recently, researchers have applied various feedback-linearization-, uncertainty-observer-, fuzzy-logic- and neural-network-based nonlinear, robust and adaptive control techniques in order to achieve synchronization of both coupled and uncoupled chaotic FHN neurons [11,15–18]. These techniques, however, are based on known FHN neuron parameter values and, therefore, their application is limited to lumped uncertainty associated with the nonlinear part of neuronal dynamics.

In invasive deep brain stimulation, an electrode is implanted in the skull of a patient in order to stimulate certain neurons. The stimulation current that arrives at two different neurons has different phase shifts according to the different path lengths from the electrode to the neurons. The amplitudes of the stimulation current vary for each neuron as well, due to the different medium losses. As these medium losses and path lengths are difficult to measure, the amplitudes and the phase shifts of the stimulation current, for neurons, are uncertain. Moreover, the parameters of FHN neurons, owing to pertinent biological restrictions, are mostly unknown. In this Letter, first, we present a coupled FHN neuron model for an uncertain stimulation current and provide a necessary condition for neuronal synchronization. Then, in order to cope with the biological restrictions, we address computationally efficient robust adaptive control for synchronization of chaotic FHN neurons with unknown neural parameters, using the knowledge of parametric bounds. We develop a linear matrix inequality (LMI)-based sufficient condition (see for example [19]) that guarantees asymptotic synchronization of FHN neurons under uncertain stimulation current amplitudes and phase shifts, and unknown neural parameters. And finally, the results of numerical simulations of coupled chaotic FHN neuron synchronization for unknown parameters and an uncertain stimulation current are provided as a

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demonstration of the effectiveness of the proposed methodology. Our main contributions are summarized below.

(1) To the best of our knowledge, this Letter investigates, first time, synchronization of FHN neurons under uncertain and different stimulation current phase shifts. Synchronization of FHN neurons with uncertain and different stimulation current amplitudes also remains rare.
(2) To the best of our knowledge, this is the first-ever report of a global robust adaptive control law for synchronization of FHN neurons with all parameters unknown.
(3) This Letter proposes a novel LMI-based robust adaptive FHN neuron synchronization strategy in which the controller parameters can be selected easily, without any tuning effort, by utilizing available LMI routines.

This Letter is organized as follows. Section 2 presents the two-coupled-FHN-neuron model for different stimulation current amplitudes and phase shifts, and presents the necessary condition for synchronization. Section 3 demonstrates the LMI-based nonlinear robust adaptive control for synchronization of uncertain coupled chaotic FHN neurons. Section 4 describes numerical simulations and presents their results. Section 5 draws conclusions.

2. Model description

Consider two coupled chaotic FHN neurons [4–6] under EES with an uncertain stimulation current given by

$$\frac{dx_1}{dt} = x_1(x_1-1)(1-rx_1) - y_1 - g(x_1-x_2) + (a_1/\omega)\cos(\omega t+\phi_1),$$
$$\frac{dy_1}{dt} = bx_1 - vy_1, \tag{1}$$
$$\frac{dx_2}{dt} = x_2(x_2-1)(1-rx_2) - y_2 - g(x_2-x_1) + (a_2/\omega)\cos(\omega t+\phi_2),$$
$$\frac{dy_2}{dt} = bx_2 - vy_2, \tag{2}$$

where x_1 and y_1 are the states of the master FHN neuron, and x_2 and y_2 are the states of the slave FHN neuron. The strength of gap junctions between the master neuron and the slave neuron is represented by g. The amplitudes of the external stimulation current for the master and slave neurons are represented by a_1 and a_2, respectively, and the phase shifts are represented by ϕ_1 and ϕ_2, respectively. Time t and angular frequency $\omega = 2\pi f$, are given as dimensionless quantities [4,10,11].

The amplitudes of the stimulation current for two neurons under EES can differ due to different medium losses. Similarly, the stimulus signal arriving at two neurons from an electrode can also have different phase shifts, due to differences in the path lengths. To consider these facts, the amplitudes (a_1, a_2) and the phase shifts (ϕ_1, ϕ_2) of the stimulation current for both coupled FHN neurons (1)–(2) are taken different. The medium losses and path lengths cannot be precisely determined, due to which reason the parameters a_1, a_2, ϕ_1, and ϕ_2 are unknown. It can easily be verified that neurons (1)–(2) are not synchronous if $a_1 \neq a_2$, and/or $\phi_1 - \phi_2 \neq 2n\pi$, for any integer n. When synchronization of the neurons occurs, we have $x_1 = x_2 = x$ and $y_1 = y_2 = y$. The synchronization errors correspondingly become $e_1 = x_1 - x_2 = 0$ and $e_2 = y_1 - y_2 = 0$. For these conditions, we conclude that

$$(a_1/\omega)\cos(\omega t+\phi_1) = (a_2/\omega)\cos(\omega t+\phi_2) \tag{3}$$

is required for synchronization of the FHN neurons (1)–(2). This implies that $a_1 = a_2$ and $\phi_1 - \phi_2 = 2n\pi$ are the necessary (but not sufficient) conditions for synchronization of the coupled FHN neurons, which shows that neurons (1)–(2) are very sensitive to the amplitudes and the phase shifts of the stimulation current. Even a small difference in these amplitudes and/or phase shifts can either desynchronize synchronous neurons or prevent synchronization of non-synchronous neurons. To address the synchronization of FHN neurons (1)–(2) under uncertain parameters and stimulation current, we use single control input u, and the overall dynamics of coupled FHN neurons becomes

$$\frac{dx_1}{dt} = x_1(x_1-1)(1-rx_1) - y_1 - g(x_1-x_2) + (a_1/\omega)\cos(\omega t+\phi_1),$$
$$\frac{dy_1}{dt} = bx_1 - vy_1, \tag{4}$$
$$\frac{dx_2}{dt} = x_2(x_2-1)(1-rx_2) - y_2 - g(x_2-x_1) + (a_2/\omega)\cos(\omega t+\phi_2) + u,$$
$$\frac{dy_2}{dt} = bx_2 - vy_2. \tag{5}$$

Assumption 1. The parameters of the FHN neurons are bounded such that

$$0 < v_{\min} \leqslant v \leqslant v_{\max}, \tag{6}$$
$$0 < b_{\min} \leqslant b \leqslant b_{\max}, \tag{7}$$
$$0 < g_{\min} \leqslant g \leqslant g_{\max}, \tag{8}$$
$$0 < r_{\min} \leqslant r \leqslant r_{\max}, \tag{9}$$

where the subscripts min and max represent the minimum and maximum values of the parameters, respectively.

Assumption 2. The parameters (a_1, a_2, ϕ_1, and ϕ_2) of the stimulation current are unknown constants.

The purpose of the present study is to develop a robust adaptive control law u for synchronization of FHN neurons (4)–(5) under Assumptions 1–2, to guarantee asymptotic convergence of the synchronization errors $e_1 = x_1 - x_2$ and $e_2 = y_1 - y_2$ to zero.

3. Robust adaptive control

In biological systems, given the infeasibility of experimental measurement and deviation from the predicted values, most model parameters are unknown. Usually, however, we have a sense of the parametric ranges that are appropriate to, and therefore can be helpful in solving biological problems. Incorporating this knowledge, makes possible the development of robust adaptive control for synchronization of FHN neurons under uncertain model and stimulation current parameters. To develop this control law, the dynamics of the synchronization errors for coupled FHN neurons (4)–(5), by employing $e_1 = x_1 - x_2$ and $e_2 = y_1 - y_2$, are written as

$$\frac{de_1}{dt} = -rx_1^3 + (1+r)x_1^2 + rx_2^3 - (1+r)x_2^2 - (2g+1)e_1 - e_2 + (a_{C\phi}/\omega)\cos\omega t - (a_{S\phi}/\omega)\sin\omega t - u,$$
$$\frac{de_2}{dt} = be_1 - ve_2, \tag{10}$$

where

$$a_{C\phi} = a_1\cos\phi_1 - a_2\cos\phi_2 \quad \text{and}$$
$$a_{S\phi} = a_1\sin\phi_1 - a_2\sin\phi_2. \tag{11}$$

Before proceeding to the design strategy, we must identify the parameters for which adaptation laws are required. We are using a single control input u, due to which adaptation laws for parameters b and v cannot be developed, so the control strategy must be sufficiently robust to handle their variations. The uncertainty in the strength of gap junctions g is associated with the linear part of the synchronization error dynamics. The robustness of control law u with respect to parameter g can be ensured straightforwardly, as is essential for reduction of the number of computations. Parameter r is associated with the nonlinear part of the synchronization error dynamics, so we can use adaptation of r for the sake of controller design procedure simplicity [20–22]. Additionally, we use two adaptation laws for stimulation current parameters $a_{C\phi}$ and $a_{S\phi}$, so as to reduce both the number of computations and the complexity of the controller design procedure, rather than using four adaptation laws for parameters a_1, a_2, ϕ_1, and ϕ_2. The proposed controller is then given by

$$u = -\hat{r}x_1^3 + (1+\hat{r})x_1^2 + \hat{r}x_2^3 - (1+\hat{r})x_2^2 + Ce_1 + (\hat{a}_{C\phi}/\omega)\cos\omega t - (\hat{a}_{S\phi}/\omega)\sin\omega t, \tag{12}$$

where $\hat{r}, \hat{a}_{C\phi}$ and $\hat{a}_{S\phi}$ are the estimates of the parameters $r, a_{C\phi}$ and $a_{S\phi}$, respectively. The adaptation laws for these parameters are given by

$$\dot{\hat{r}} = pe_1\left(-x_1^3 + x_1^2 + x_2^3 - x_2^2\right)/q_1, \quad p > 0,\ q_1 > 0, \tag{13}$$

$$\dot{\hat{a}}_{C\phi} = p(e_1\cos\omega t)/(\omega q_2), \quad q_2 > 0, \tag{14}$$

$$\dot{\hat{a}}_{S\phi} = -p(e_1\sin\omega t)/(\omega q_3), \quad q_3 > 0. \tag{15}$$

Note that the control law (12) and the adaptation laws (13)–(15), in contrast to the conventional techniques [11,15–18], do not require measurements of neural states y_1 and y_2. Now we provide an LMI-based sufficient condition for asymptotic synchronization of the FHN neurons.

Theorem 1. *Consider the FHN neurons (4)–(5) with the synchronization error dynamics (10)–(11) satisfying Assumptions 1–2. Suppose that the LMI's*

$$z > 0, \qquad p > 0, \qquad \varepsilon > 0, \tag{16}$$

$$\begin{bmatrix} -z-p & -p/2 & \varepsilon\sqrt{b_{\max}/2} \\ * & -v_{\min} & \sqrt{b_{\max}/2} \\ * & * & -\varepsilon \end{bmatrix} < 0 \tag{17}$$

are satisfied. Then, the nonlinear control law (12) along with the adaptation laws (13)–(15) ensures:

(i) *synchronization of the coupled FHN neurons with asymptotic convergence of synchronization errors e_1 and e_2 to zero;*
(ii) *convergence of adaptive parameters $\hat{r}$, $\hat{a}_{C\phi}$ and $\hat{a}_{S\phi}$ to r^*, $a_{C\phi}$ and $a_{S\phi}$, respectively, where r^* is a constant value.*

The controller parameter C is given by $C = z/p$.

Proof. Incorporating (12) into (10), the error dynamics become

$$\frac{de_1}{dt} = (r-\hat{r})\left(-x_1^3 + x_1^2 + x_2^3 - x_2^2\right) - (C+1)e_1 - e_2 - 2ge_1 + \left((a_{C\phi} - \hat{a}_{C\phi})/\omega\right)\cos\omega t - \left((a_{S\phi} - \hat{a}_{S\phi})/\omega\right)\sin\omega t,$$

$$\frac{de_2}{dt} = be_1 - ve_2. \tag{18}$$

Constructing the Lyapunov function (see for example [23–25])

$$E = (1/2)\left(pe_1^2 + e_2^2 + q_1(r-\hat{r})^2 + q_2(a_{C\phi} - \hat{a}_{C\phi})^2 + q_3(a_{S\phi} - \hat{a}_{S\phi})^2\right), \tag{19}$$

with $p > 0$, $q_1 > 0$, $q_2 > 0$, and $q_3 > 0$, the derivative of (19) is given by

$$\dot{E} = pe_1\dot{e}_1 + e_2\dot{e}_2 - q_1(r-\hat{r})\dot{\hat{r}} - q_2(a_{C\phi} - \hat{a}_{C\phi})\dot{\hat{a}}_{C\phi} - q_3(a_{S\phi} - \hat{a}_{S\phi})\dot{\hat{a}}_{S\phi}. \tag{20}$$

Incorporating (18) into (20), we obtain

$$\dot{E} = pe_1(r-\hat{r})\left(-x_1^3 + x_1^2 + x_2^3 - x_2^2\right) - p(C+1)e_1^2 - pe_1e_2 - 2gpe_1^2 + pe_1\left((a_{C\phi} - \hat{a}_{C\phi})/\omega\right)\cos\omega t - pe_1\left((a_{S\phi} - \hat{a}_{S\phi})/\omega\right)\sin\omega t + be_1e_2 - ve_2^2 - q_1(r-\hat{r})\dot{\hat{r}} - q_2(a_{C\phi} - \hat{a}_{C\phi})\dot{\hat{a}}_{C\phi} - q_3(a_{S\phi} - \hat{a}_{S\phi})\dot{\hat{a}}_{S\phi}. \tag{21}$$

Using the adaptation laws (13)–(15) into (21), we get

$$\dot{E} = -p(C+1)e_1^2 - pe_1e_2 - 2gpe_1^2 + be_1e_2 - ve_2^2, \tag{22}$$

$$\dot{E} \leqslant -p(C+1)e_1^2 - pe_1e_2 - 2gpe_1^2 + be_1e_2 - v_{\min}e_2^2, \tag{23}$$

$$\dot{E} < -p(C+1)e_1^2 - pe_1e_2 + be_1e_2 - v_{\min}e_2^2. \tag{24}$$

For any $\varepsilon > 0$, we get the following inequality [26].

$$be_1e_2 \leqslant (1/2)b\left(\varepsilon e_1^2 + \varepsilon^{-1}e_2^2\right) \leqslant (1/2)b_{\max}\left(\varepsilon e_1^2 + \varepsilon^{-1}e_2^2\right). \tag{25}$$

Using (24) and (25), we obtain

$$\dot{E} < -p(C+1)e_1^2 - pe_1e_2 + (1/2)b_{\max}\left(\varepsilon e_1^2 + \varepsilon^{-1}e_2^2\right) - v_{\min}e_2^2. \tag{26}$$

For asymptotic convergence of the synchronization errors, $\dot{E} < 0$. Hence

$$\dot{E} < e^T\Phi e < 0, \tag{27}$$

where

$$e = [\,e_1 \quad e_2\,]^T, \tag{28}$$

$$\Phi = \begin{bmatrix} -p(C+1) + (1/2)b_{\max}\varepsilon & -p/2 \\ * & -v_{\min} + (1/2)b_{\max}\varepsilon^{-1} \end{bmatrix} < 0. \tag{29}$$

By applying the Schur complement [27–29] to the inequality (29) and using $z = pC$, we obtain the LMI's of (16)–(17). Thus, the asymptotic convergence of the synchronization errors (e_1 and e_2) to zero is ensured, which completes the proof of statement (i) in Theorem 1. In the steady state, the synchronization errors and the states of neurons satisfy

$$[\,\dot{e}_1 \quad \dot{e}_2\,] = [\,0 \quad 0\,], \qquad [\,e_1 \quad e_2\,] = [\,0 \quad 0\,], \tag{30}$$

$$[\,x_1 \quad y_1\,] = [\,x_2 \quad y_2\,]. \tag{31}$$

Using $e_1 = 0$ in (13)–(15), $\dot{\hat{r}} = 0$, $\dot{\hat{a}}_{C\phi} = 0$, and $\dot{\hat{a}}_{S\phi} = 0$ are satisfied in the steady state. This further implies that

$$\hat{r} = r^*, \qquad \hat{a}_{C\phi} = \hat{a}^*_{C\phi}, \quad \text{and} \quad \hat{a}_{S\phi} = \hat{a}^*_{S\phi} \tag{32}$$

are satisfied in the steady state, where r^*, $\hat{a}^*_{C\phi}$ and $\hat{a}^*_{S\phi}$ are the constant steady state values. Now putting the steady state conditions from (30)–(32) into (18), we obtain

$$\left((a_{C\phi} - \hat{a}^*_{C\phi})/\omega\right)\cos\omega t - \left((a_{S\phi} - \hat{a}^*_{S\phi})/\omega\right)\sin\omega t = 0, \tag{33}$$

which can only be true if $\hat{a}^*_{C\phi} = a_{C\phi}$ and $\hat{a}^*_{S\phi} = a_{S\phi}$, because the stimulus frequency ω cannot be infinity. Hence the steady state values of the adaptive parameters $\hat{a}_{C\phi}$ and $\hat{a}_{S\phi}$ are equal to $a_{C\phi}$

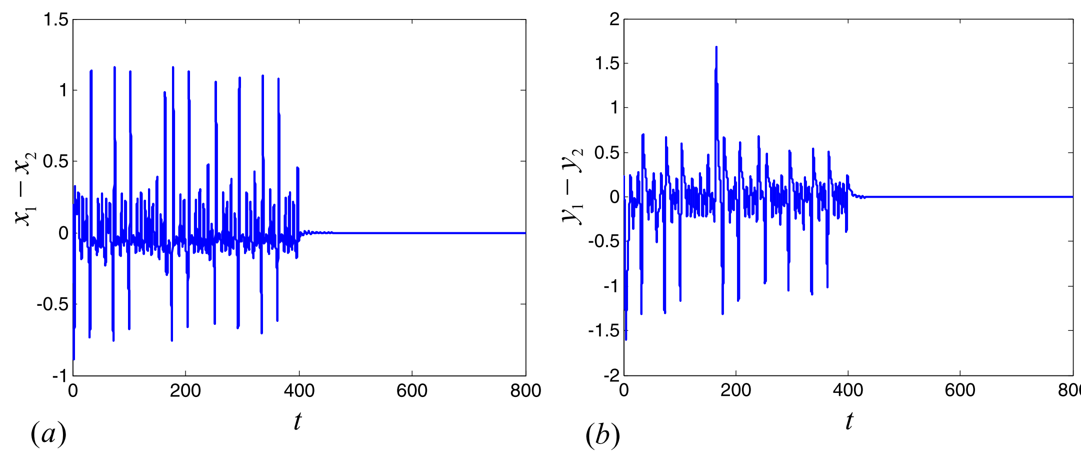

Fig. 1. Synchronization error plots for the coupled chaotic uncertain FHN neurons under EES. The controller was activated at $t = 400$. Both synchronization errors converge to zero by application of the robust adaptive controller. (a) Synchronization error $e_1 = x_1 - x_2$, (b) synchronization error $e_2 = y_1 - y_2$.

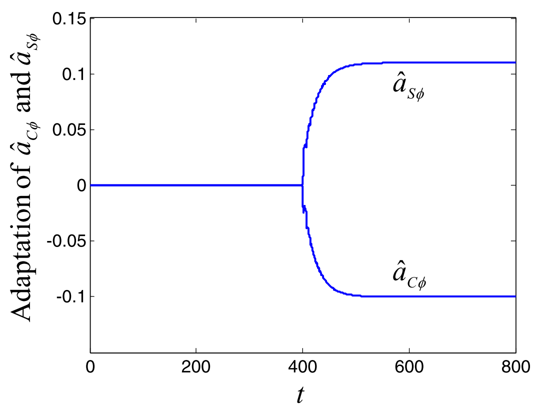

Fig. 2. Convergence of the adaptive parameters $\hat{a}_{C\phi}$ and $\hat{a}_{S\phi}$ to the stimulation parameters $a_{C\phi}$ and $a_{S\phi}$, respectively, by application of the robust adaptive control.

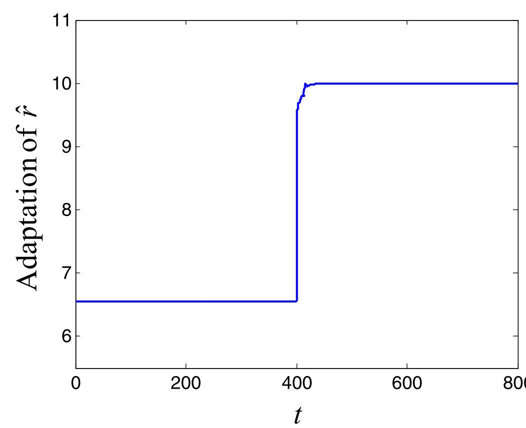

Fig. 3. Convergence of the adaptive parameter $\hat{r}$ to a constant value by application of the robust adaptive control.

and $a_{S\phi}$, respectively. Thus, our adaptation laws guarantee precise estimation of the unknown parameters related to the stimulation current. This completes the proof of statement (ii) in Theorem 1. It is worth noting that convergence of adaptive parameters $\hat{a}_{C\phi}$ and $\hat{a}_{S\phi}$ to stimulation current parameters $a_{C\phi}$ and $a_{S\phi}$ is ensured by incorporating the steady state knowledge into the adaptation laws and the synchronization error dynamics, but is not guaranteed by the Lyapunov method. □

It is often necessary to minimize control efforts by minimizing controller gain [25]. In the present scenario, the control efforts can be minimized by minimizing controller parameter C. For this purpose, we can transform the LMI's (16)–(17) into the following optimization problem.

$$\min C,$$

subject to

$$\varepsilon > 0,\ p > 0,\quad \begin{bmatrix} -p(C+1) & -p/2 & \varepsilon\sqrt{b_{\max}/2} \\ * & -v_{\min} & \sqrt{b_{\max}/2} \\ * & * & -\varepsilon \end{bmatrix} < 0. \tag{34}$$

The inequality (34) is a bilinear matrix inequality, which can be treated as an LMI for a selection of scalar p (say, for example, $p = 1$). However, the main results proposed in Theorem 1 do not require any parameter tuning to find the controller parameter C.

4. Simulation results

For validation of the proposed methodology, we choose the model parameters as $r = 10$, $g = 0.05$, $f = 0.145$, $b = 1$, $v = 0.07$, $a_1 = 0.1$, $a_2 = 0.11$, $\phi_1 = \pi$, and $\phi_2 = 1.5\pi$, for which the FHN neurons (1)–(2) exhibit chaotic behavior. The initial conditions are taken as $x_1(0) = 0.1$, $y_1(0) = 0.1$, $x_2(0) = -0.1$, $y_2(0) = -0.1$, $\hat{a}_{C\phi}(0) = 0$, $\hat{a}_{S\phi}(0) = 0$, and $\hat{r}(0) = 6.56$. By solving Theorem 1, the controller parameters $C = 6.2009$, $p = 1$, $q_1 = 10^{-5}$, $q_2 = 2$, and $q_3 = 2$ are obtained for the parametric ranges $v \in [0.05, 0.1]$ and $b \in [0.9, 1.1]$. Fig. 1 shows the synchronization error plots obtained with the proposed control law. The controller is applied at $t = 400$. It is clear that, using the controller, both synchronization errors are converging to zero. The plots for the adaptive parameters $\hat{a}_{C\phi}$ and $\hat{a}_{S\phi}$ are shown in Fig. 2. Both parameters are converging to $a_{C\phi} = -0.1$ and $a_{S\phi} = 0.11$, respectively. Fig. 3 plots the adaptive parameter $\hat{r}$, which converges to a constant value by application of the proposed controller. The FHN neurons thus are synchronized by means of the robust adaptive control methodology.

5. Conclusions

This Letter addresses the synchronization of two coupled chaotic FHN neurons for unknown parameters and uncertain stimulation current amplitudes and phase shifts. By incorporating knowledge of parametric bounds, an LMI-based nonlinear robust adaptive control law, which guarantees the asymptotic convergence of the synchronization errors to zero, was formulated. Additionally, our strategy guarantees precise adaptation of external stimulation current parameters. The proposed scheme was applied to the synchronization of coupled FHN neurons by providing simulation results.

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